Derivative

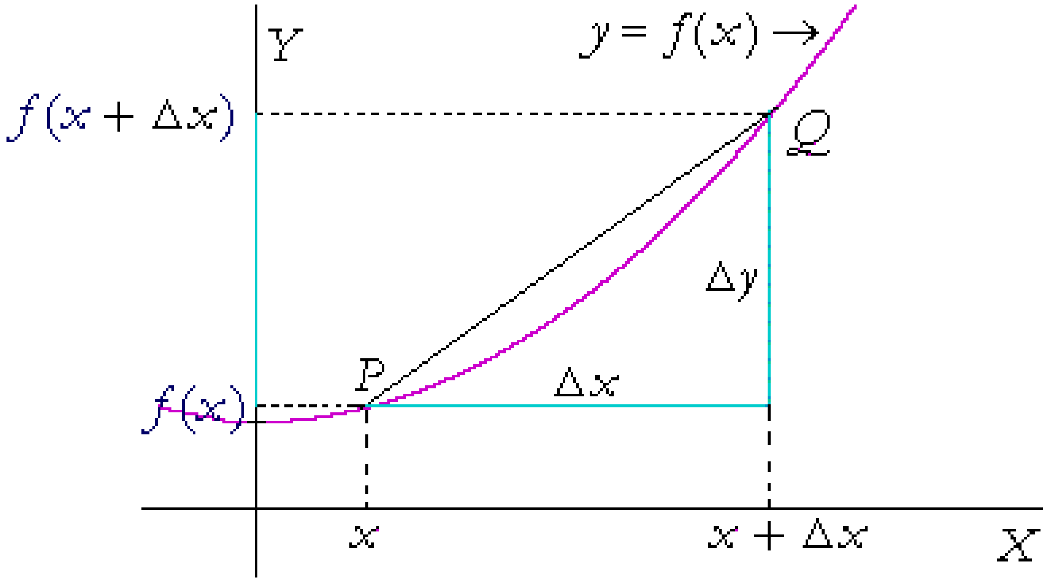
In Mathematics, the derivative is a way to show rate of change of a function y (let, $y = f(x)$) with respect to some other variable (x).

It is denoted as $\frac{dy}{dx} = \frac{d}{dx}y = \frac{d}{dx}f(x) = f'(x) = y_1$

$\frac{dy}{dx}$ is read as – “d dx of y”.

Remember that -

1. $\frac{dy}{dx} \neq dy \div dx$
2. d is an operator. $dy \neq d \times y$



Let $y = f(x)$ be a continuous function and the co-ordinate of point P on the graph be $(x, f(x))$.

Let x now change by an amount Δx. The new x co-ordinate is $x + \Delta x$.

As $y = f(x)$ so the value of y will depend upon x. So, any change (Δx) in the value of x, will result a change in the value of y. Let it be Δy. So, the new value of y is $y + \Delta y = f(x + \Delta x)$. The point Q shows the new position.

The quotient $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$ is known as Newton quotient, or the difference quotient.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

So, derivative of y with respect to x is the rate of change of y with respect to infinitesimal change of x.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ where } \Delta x = h$$

$$\left(\frac{dy}{dx}\right)_{x=a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

[Let $h = x - a$. $h \to 0 \Rightarrow x - a \to 0 \Rightarrow x \to a$]

Formula Table: <u>*csc*</u> is the short form of <u>*cosec*</u>.

$\frac{d}{dx}(x^n) = nx^{n-1}$ where n rational	$\frac{d}{dx}(\sin x) = \cos x$ [x in radian]
$\frac{d}{dx}(\cos x) = -\sin x$ [x in radian]	$\frac{d}{dx}(\tan x) = \sec^2 x$ [x in radian]
$\frac{d}{dx}(\cot x) = -csc^2\, x$ [x in radian]	$\frac{d}{dx}(\sec x) = \sec x \tan x$ [x in radian]
$\frac{d}{dx}(\csc x) = -\csc x \cot x$ [x in radian]	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(a^x) = a^x \log_e a$ $[a > 0]$	$\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $[x \neq 0]$
$\frac{d}{dx}(c) = 0$ [c constant]	$\frac{d}{dx}\{c \cdot f(x)\} = c \cdot \frac{d}{dx}\{f(x)\}$ [c constant]
$\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$	$\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$
$\frac{d}{dx}(uvw) = vw \cdot \frac{du}{dx} + uw \cdot \frac{dv}{dx} + uv \cdot \frac{dw}{dx}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $(\lvert x \rvert < 1)$	$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ $(\lvert x \rvert < 1)$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ $(-\infty < x < \infty)$	$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$ $(-\infty < x < \infty)$

$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}} \quad (\lvert x\rvert > 1)$	$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \quad (\lvert x\rvert > 1)$

Chain rule:

If $y = f(u)$ and $u = \phi(x)$ then

$$\frac{dy}{dx} = \frac{d}{dx}y = \frac{d}{dx}\{f(u)\} = \frac{d}{du}f(u)\cdot\frac{du}{dx} = \frac{df}{du}\cdot\frac{d\phi}{dx} = f'\phi'$$

Note: As y is a function of u, so we can't make derivative of y w.r.t. x directly.

Derivative from parametric equation:

If $y = f(t)$ and $x = f(t)$, then

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} \text{ (chain rule)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{f\prime(t)}{\phi\prime(t)}$$

Example – 1: **Find the derivate of following with respect to (w.r.t) x or find $\frac{dy}{dx}$**

i. $\log\left(\tan\frac{x}{2}\right)$

ii. $\frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x-3)$

iii. $\log_{\sin x}\sec x + 10^{x^2}$

iv. $x^{\sin x}$

v. x^{e^x}

vi. $(\log x)^{\cos x}$

vii. $10^x \cdot x^{10}$

viii. $x^x + (\sin x)^x$

ix. $(\tan x)^{\cot x} + (\cot x)^{\tan x}$

x. $[(\tan x)^{\tan x}]^{\tan x}$ at $x = \frac{\pi}{4}$

xi. $x^y = y^x$

xii. $x^y \cdot y^x = 1$

xiii. $x^3y^4 = (x+y)^7$

xiv. $x^y + y^x = a$

xv. $y^x = e^{y-x}$

xvi. $(\cos x)^y = (\sin y)^x$

xvii. $xy = \tan(x+y)$

xviii. $y = e^{\frac{y}{x}}$

xix. $ye^y = x$

xx. $x^{\sin y} + y^{\sin x} = 1$

xxi. $y = x^{x^{x^{\cdots\infty}}}$

xxii. $y = x + \frac{1}{x+\frac{1}{x+\cdots\infty}}$

2.

i. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

ii. If $y = \frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$ then show that $\frac{dy}{d\theta} + \frac{1}{1-\cos\theta} = 0$

iii. If $\sin y = x\sin(a+y)$ then prove that $\frac{dy}{dx} = \frac{\sin a}{1-2x\cos a+x^2}$

iv. If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ then show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

v. If $y = 1 + \frac{a}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$ then show that $\frac{dy}{dx} = \frac{y}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$

vi. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

vii. If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$ then show that $f'(0) = \left(2\log\frac{a}{b} + \frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$

Solutions of 1:

Let, $\log\left(\tan\frac{x}{2}\right) = y$.

$\therefore \frac{dy}{dx} = ?$

[Let, $\tan\frac{x}{2} = u$]

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\log\left(\tan\frac{x}{2}\right)\right\}$$

$$= \frac{d}{dx}(\log u)$$

$$= \frac{d}{du}(\log u)\cdot\frac{du}{dx} \quad \text{[Chain rule]}$$

$$= \frac{1}{u}\cdot\frac{d}{dx}\left(\tan\frac{x}{2}\right) \quad \left[\text{Let, } \frac{x}{2} = v\right]$$

$$= \frac{1}{u}\ \frac{d}{dx}(\tan v)$$

$$= \frac{1}{u}\ \frac{d}{dv}(\tan v)\cdot\frac{dv}{dx} \quad \text{[Chain rule]}$$

$$= \frac{1}{u}\sec^2 v\cdot\frac{d}{dx}\left(\frac{1}{2}x\right) \quad \text{[Putting value of } v\text{]}$$

$$= \frac{1}{\tan\frac{x}{2}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}\cdot\frac{d}{dx}(x)$$

$$= \frac{1}{2}\,\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\cdot\frac{1}{\cos^2\frac{x}{2}}\cdot 1 \quad \left[\because \frac{d}{dx}(x) = 1\right]$$

$$= \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{\sin x} = \underline{\underline{\operatorname{cosec} x}}$$

ANS.

$$\frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x-3)$$

GOOD BOY
PAGE NO:
DATE:

$$y = \frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x-3)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5x}{\sqrt[3]{1-x^2}}\right) + \frac{d}{dx}\left\{\sin^2(2x-3)\right\}$$

Now,

$$\frac{d}{dx}\left(\frac{5x}{\sqrt[3]{1-x^2}}\right) = \frac{d}{dx}\left\{5x.\left(1-x^2\right)^{-1/3}\right\}$$

$$= 5x.\left(-\frac{1}{3}\right)\left(1-x^2\right)^{-\frac{1}{3}-1}.\frac{d}{dx}\left(1-x^2\right) + \left(1-x^2\right)^{-1/3}\frac{d}{dx}(5x)$$

$$= -\frac{5x}{3}\left(1-x^2\right)^{-4/3}.(-2x) + \left(1-x^2\right)^{-1/3}.5$$

$$= \frac{10x^2}{3\left(1-x^2\right)^{4/3}} + \frac{5}{\left(1-x^2\right)^{1/3}}$$

$$= \frac{10x^2 + 3\times5\times(1-x^2)}{3\left(1-x^2\right)^{4/3}} = \frac{10x^2+15-15x^2}{3\left(1-x^2\right)^{4/3}}$$

$$= \frac{15-5x^2}{3\left(1-x^2\right)^{4/3}} = \frac{5(3-x^2)}{3\left(1-x^2\right)^{4/3}}$$

Now, $\frac{d}{dx}\left\{\sin^2(2x-3)\right\}$

$$= 2.\sin(2x-3)\cos(2x-3).\frac{d}{dx}(2x-3)$$

$$= 2\sin(2x-3)\cos(2x-3).2$$

$$= 2\sin(4x-6)$$

$$\therefore \frac{dy}{dx} = \frac{5(3-x^2)}{3\left(1-x^2\right)^{4/3}} + 2\sin(4x-6) \quad \text{(Ans)}$$

$$y = \log_{\sin x} \sec x + 10^{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\log_{\sin x} \sec x\right) + 10^{x^2}$$

$$\left[\log_{\sin x} \sec x = \frac{\log_e \sec x}{\log_e \sin x}\right]$$

$$\therefore \frac{d}{dx}\left(\log_{\sin x} \sec x\right) = \frac{d}{dx}\left(\frac{\log_e \sec x}{\log_e \sin x}\right)$$

$$= \frac{\log_e \sin x \cdot \frac{d}{dx}(\log_e \sec x) - \log_e \sec x \cdot \frac{d}{dx}(\log_e \sin x)}{(\log \sin x)^2}$$

$$= \frac{\log \sin x \cdot \frac{\sec x \cdot \tan x}{\sec x} - \log_e \sec x \cdot \frac{\cos x}{\sin x}}{(\log \sin x)^2}$$

$$= \frac{\tan x \log(\sin x) - \cot x \log(\sec x)}{(\log \sin x)^2}$$

And, $\frac{d}{dx}\left(10^{x^2}\right) = 10^{x^2} \log_e 10 \cdot \frac{d}{dx}(x^2)$

$$= 2x \cdot 10^{x^2} \cdot \log_e 10$$

$$\therefore \frac{dy}{dx} = \frac{\tan x \log(\sin x) - \cot x \log(\sec x)}{(\log \sin x)^2} + 2x \cdot 10^{x^2} \log_e 10$$

Ans.

GOOD BOY
PAGE NO:
DATE:

$y = x^{\sin x}$

Taking log on both sides,

$\log y = \log x^{\sin x}$

or, $\log y = \sin x \log x$

Diff. both sides w.r.t. x, We get,

$\frac{d}{dx}(\log y) = \frac{d}{dx}(\sin x . \log x)$

or, $\frac{d}{dy}(\log y) \frac{dy}{dx} = \sin x . \frac{d}{dx}(\log x) + \log x . \frac{d}{dx}(\sin x)$

or, $\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \log x . \cos x$

or, $\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x . \log x\right]$ (Ans).

$y = x^{e^x}$

Taking log, $\log y = \log x^{e^x} = e^x \log x$

Taking derivative, w.r.t. x,

$\frac{d}{dx}(\log y) = \frac{d}{dx}(e^x . \log x)$

or, $\frac{1}{y}\frac{dy}{dx} = e^x . \frac{d}{dx}(\log x) + \log x . \frac{d}{dx}(e^x)$

or, $\frac{dy}{dx} = x^{e^x} . e^x \left(\frac{1}{x} + \log x\right)$

$(\log x)^{\cos x}$

$$y = (\log x)^{\cos x}$$

Taking log, $\log y = \log(\log x)^{\cos x}$

$$\log y = \cos x \cdot \log(\log x)$$

Differentiate both sides, w.r.t. x, we get,

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\left\{\cos x \cdot \log(\log x)\right\}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}\cos x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x\log x} + \log(\log x)\cdot(-\sin x)$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x \cdot \log(\log x)\right]$$

$y = 10^x \cdot x^{10}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(10^x \cdot x^{10}) = 10^x \cdot \frac{d}{dx}(x^{10}) + x^{10} \cdot \frac{d}{dx}(10^x)$

or, $\frac{dy}{dx} = 10^x \cdot 10x^9 + x^{10} \cdot 10^x \cdot \log_e 10$

$= 10^x \cdot x^9 (10 + x \log 10)$. Ans

GOOD BOY
PAGE NO:
DATE:

$y = x^x + (\sin x)^x$

$\frac{dy}{dx} = \frac{d}{dx}(x^x) + \frac{d}{dx}(\sin x)^x$

Now, $\frac{d}{dx}(x^x) = ?$

let, $x^x = u \quad \therefore \frac{du}{dx} = ?$

$\log u = \log x^x = x \log x.$

Diff. w.r.t x, $\frac{d}{du}(\log u)\frac{du}{dx} = \frac{d}{dx}(x \log x)$

or, $\frac{1}{u}\frac{du}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$

or, $\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$

or, $\frac{du}{dx} = x^x(1 + \log x)$.

Now, $\frac{d}{dx}(\sin x)^x = ?$

let, $(\sin x)^x = v$. $\therefore \frac{dv}{dx} = ?$

Taking log,
$\log v = x \log(\sin x)$

Diff. w.r.t. x, $\frac{d}{dx}(\log v) = \frac{d}{dx}(x \log \sin x)$

or, $\frac{d}{dv}(\log v) \cdot \frac{dv}{dx} = x \cdot \frac{d}{dx}(\log \sin x) + \log \sin x \cdot \frac{d}{dx}(x)$

or, $\frac{1}{v}\frac{dv}{dx} = x \frac{\cos x}{\sin x} + \log(\sin x)$

or, $\frac{dv}{dx} = (\sin x)^x(x \cot x + \log \sin x)$

$\frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x(x \cot x + \log \sin x)$ Ans

$y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left\{(\tan x)^{\cot x}\right\} + \frac{d}{dx}\left\{(\cot x)^{\tan x}\right\}$

$= \frac{du}{dx} + \frac{dv}{dx}$, where,

$u = (\tan x)^{\cot x}$ & $v = (\cot x)^{\tan x}$,

Now, $u = (\tan x)^{\cot x}$

$\therefore \log u = \cot x \log(\tan x)$

$\therefore \frac{d}{dx}(\log u) = \frac{d}{dx}\left\{\cot x \cdot \log(\tan x)\right\}$

or, $\frac{d}{du}(\log u)\frac{du}{dx} = \cot x \cdot \frac{d}{dx}\log(\tan x) + \log(\tan x) \cdot \frac{d}{dx}(\cot x)$

or, $\frac{1}{u}\frac{du}{dx} = \cot x \cdot \frac{\sec^2 x}{\tan x} + \log(\tan x) \cdot (-\operatorname{cosec}^2 x)$

or, $\frac{1}{u}\frac{du}{dx} = \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} - \operatorname{cosec}^2 x \cdot \log(\tan x)$

or, $\frac{du}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x\,(1 - \log \tan x)\right]$

Now, $v = (\cot x)^{\tan x}$

$\therefore \log v = \tan x \cdot \log(\cot x)$

Differentiating both sides, w.r.t. x,

$\frac{d}{dx}(\log v) = \frac{d}{dx}\left\{\tan x \cdot \log(\cot x)\right\}$

or, $\frac{d}{dv}(\log v)\frac{dv}{dx} = \tan x \frac{d}{dx}\log(\cot x) + \log(\cot x) \cdot \frac{d}{dx}(\tan x)$

DATE:

or, $\frac{1}{v}\frac{dv}{dx} = \tan x \cdot \frac{(-\operatorname{cosec}^2 x)}{\cot x} + \log(\cot x) \cdot \sec^2 x$

or, $\frac{dv}{dx} = (\cot x)^{\tan x}\left[\frac{\sin x}{\cos x} \cdot \frac{\sin x}{(-\sin^2 x)\cdot\cos x} + \log(\cot x) \cdot \sec^2 x\right]$

or, $\frac{dv}{dx} = (\cot x)^{\tan x}\left[\sec^2 x\left(\log \cot x - 1\right)\right]$

$\therefore \frac{dy}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x\left(1 - \log \tan x\right)\right]$

$+ (\cot x)^{\tan x}\left[\sec^2 x\left(\log \cot x - 1\right)\right]$.

Ans

$$y = \left\{(\tan x)^{\tan x}\right\}^{\tan x} = \tan x^{\tan^2 x}$$

Taking log, $\log y = \tan^2 x \cdot \log(\tan x)$.

Diff. both sides, w.r.t. x, we get,

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\left\{\tan^2 x \cdot \log(\tan x)\right\}$$

or, $$\frac{d}{dy}(\log y)\cdot\frac{dy}{dx} = \tan^2 x \cdot \frac{d}{dx}\log(\tan x) + \log(\tan x)\cdot\frac{d}{dx}(\tan^2 x)$$

or, $$\frac{1}{y}\frac{dy}{dx} = \tan^2 x \cdot \frac{\sec^2 x}{\tan x} + \log(\tan x)\cdot 2\tan x \cdot \sec^2 x$$

or, $$\frac{1}{y}\frac{dy}{dx} = \tan x \cdot \sec^2 x + 2\log(\tan x)\cdot\tan x\cdot\sec^2 x$$

or, $$\frac{dy}{dx} = y\cdot\tan x\cdot\sec^2 x\,(1 + 2\log\tan x)$$

or, $$\frac{dy}{dx} = \left\{(\tan x)^{\tan x}\right\}^{\tan x}\cdot\tan x\cdot\sec^2 x\,(1 + 2\log\tan x)$$

At, $x = \pi/4$,

$$\left(\frac{dy}{dx}\right)\Bigg|_{x=\frac{\pi}{4}} = 2(1 + 2\times 0) = 2 \quad \text{(Ans)}.$$

$x^y = y^x$

Taking log, $y \log x = x \log y$

Diff, $\frac{d}{dx}(y \log x) = \frac{d}{dx}(x \log y)$

or, $\frac{dy}{dx} \cdot \log x + y \cdot \frac{d}{dx}(\log x) = \log y \cdot 1 + x \cdot \frac{d}{dx}(\log y)$

or, $\frac{dy}{dx} \cdot \log x + \frac{y}{x} = \log y + x \cdot \frac{d}{dy}(\log y) \cdot \frac{dy}{dx}$

or, $\log x \cdot \frac{dy}{dx} + \frac{y}{x} = \log y + \frac{x}{y} \cdot \frac{dy}{dx}$

or, $\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}$

or, $\frac{dy}{dx} = \frac{\log y - y/x}{\log x - x/y} = \frac{\frac{x \log y - y}{x}}{\frac{y \log x - x}{y}}$

or, $\frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$ Ans

$x^y \cdot y^x = 1$

GOOD BOY
PAGE NO:
DATE:

Taking log, $y \log x + x \log y = 0$ ----- (i)

Diff $\frac{d}{dx}(y \log x) + \frac{d}{dx}(x \log y) = 0$

or, $\frac{dy}{dx} \cdot \log x + y \cdot \frac{d}{dx}(\log x) + x \cdot \frac{d}{dx}(\log y) + \log y \cdot 1 = 0$

or, $\frac{dy}{dx} \cdot \log x + \frac{y}{x} + x \cdot \frac{d}{dy}(\log y) \cdot \frac{dy}{dx} + \log y = 0$

or, $\frac{dy}{dx} \cdot \log x + \frac{y}{x} + \frac{x}{y} \frac{dy}{dx} + \log y = 0$

or, $\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = -\left(\log y + \frac{y}{x}\right)$

or, $\frac{dy}{dx} = -\frac{\log y + \frac{y}{x}}{\log x + \frac{x}{y}} = -\frac{y(x \log y + y)}{x(y \log x + x)}$

or, $\frac{dy}{dx} = -\frac{y(-y \log x + y)}{x(-x \log y + x)}$ [from (i)]

or, $\frac{dy}{dx} = -\frac{y^2(1 - \log x)}{x^2(1 - \log y)}$

$x^3y^4 = (x+y)^7$

$$x^3 \cdot y^4 = (x+y)^7$$

Taking log, $3\log x + 4\log y = 7\log(x+y)$

Diff, $\dfrac{3}{x} + \dfrac{4}{y}\dfrac{dy}{dx} = \dfrac{7}{x+y}\left(1 + \dfrac{dy}{dx}\right)$

or, $\dfrac{dy}{dx}\left(\dfrac{4}{y} - \dfrac{7}{x+y}\right) = \dfrac{7}{x+y} - \dfrac{3}{x}$

or, $\dfrac{dy}{dx} \cdot \dfrac{(4x+4y-7y)}{y(x+y)} = \dfrac{7x-3x-3y}{x(x+y)}$

or, $\dfrac{dy}{dx} \cdot \dfrac{(4x-3y)}{y(x+y)} = \dfrac{4x-3y}{x(x+y)}$

or, $\dfrac{dy}{dx} = \dfrac{y}{x}$ (Ans)

$x^y + y^x = a$

Diff. w.r.t. x, $\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = \frac{d}{dx}(a)$

or, $\frac{du}{dx} + \frac{dv}{dx} = 0$ —— (1). [As, a = constant, let, $x^y = u$, $y^x = v$]

$u = x^y$

or, $\log u = y \log x$

$\therefore \frac{d}{dx}(\log u) = \frac{d}{dx}(y \log x) = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$

or, $\frac{d}{du}(\log u) \cdot \frac{du}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$

or, $\frac{du}{dx} = x^y\left(\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right)$

$v = y^x$

or, $\log v = x \log y$

$\therefore \frac{d}{dx}(\log v) = \frac{d}{dx}(x \log y)$

or, $\frac{d}{dv}(\log v) \cdot \frac{dv}{dx} = x \cdot \frac{d}{dx}(\log y) + \log y \cdot \frac{d}{dx}(x)$

PAGE NO.
DATE:

or, $\frac{1}{v} \cdot \frac{dv}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y$

or, $\frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right)$

From (i),

$x^y \left(\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) + y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0$

or, $y \cdot x^{y-1} + x^y \log x \cdot \frac{dy}{dx} + x \cdot y^{x-1} \frac{dy}{dx} + y^x \log y = 0$

or, $\frac{dy}{dx} \left(x^y \log x + x y^{x-1} \right) = -\left(y x^{y-1} + y^x \log y \right)$

or, $\frac{dy}{dx} = -\frac{y x^{y-1} + y^x \log y}{x y^{x-1} + x^y \log x}$

$$y^x = e^{y-x}$$

$y^x = e^{y-x}$

Taking log, $x \log y = (y-x) \log_e e$

or, $x \log y = y - x$, $[\because \log_e e = 1]$ — (i)

Diff, $\frac{d}{dx}(x \log y) = \frac{dy}{dx} - \frac{dx}{dx}$

or, $\frac{x}{y}\frac{dy}{dx} + \log y \cdot 1 = \frac{dy}{dx} - 1$

or, $\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -(1 + \log y)$

or, $\frac{dy}{dx}\left(\frac{x-y}{y}\right) = -(1 + \log y)$

GOOD BOY
PAGE NO:
DATE:

or, $\frac{dy}{dx} = \frac{(1+\log y)\, y}{y-x}$

or,

$\frac{dy}{dx} = \frac{y(\log e + \log y)}{x \log y}$ [using (i)]

or, $\frac{dy}{dx} = \frac{y}{x} \cdot \frac{\log ey}{\log y}$

or, $\frac{dy}{dx} = \frac{(\log ey)^2}{\log y}$

[From (i),
$y - x = x \log y$
or, $\frac{y}{x} - 1 = \log y$
or, $\frac{y}{x} = 1 + \log y$
$= \log ey$]

$$(\cos x)^y = (\sin y)^x$$

$$(\cos x)^y = (\sin y)^x$$

Taking log, $y \cdot \log(\cos x) = x \log(\sin y)$

Diff both sides, w.r.t. x, we get,

$$\frac{d}{dx}\{y \log(\cos x)\} = \frac{d}{dx}\{x \log(\sin y)\}$$

or, $y \cdot \frac{d}{dx}\log(\cos x) + \log(\cos x)\frac{dy}{dx} =$

$$\log(\sin y) \cdot \frac{dx}{dx} + x\frac{d}{dx}\log(\sin y)$$

or, $y \cdot \frac{(-\sin x)}{\cos x} + \log(\cos x)\frac{dy}{dx} = \log(\sin y)$

$$+ x \cdot \frac{\cos y}{\sin y} \frac{dy}{dx}$$

or, $-y\tan x + \log(\cos x) \cdot \frac{dy}{dx} = \log(\sin y)$

$$+ x\cot y \frac{dy}{dx}$$

or, $\frac{dy}{dx}(\log\cos x - x\cot y) = y\tan x + \log \sin y$

or, $\frac{dy}{dx} = \frac{y\tan x + \log(\sin y)}{\log(\cos x) - x\cot y}$ [Ans]

$$xy = \tan(x+y)$$

$$xy = \tan(x+y)$$

Diff, w.r.t. x,

$$x \cdot \frac{dy}{dx} + y \cdot 1 = \sec^2(x+y) \cdot \left(1 + \frac{dy}{dx}\right)$$

or, $\frac{dy}{dx}\left\{x - \sec^2(x+y)\right\} = \sec^2(x+y) - y$

or, $\frac{dy}{dx} = \frac{1+x^2y^2-y}{x-1-x^2y^2} = \frac{y-1-x^2y^2}{1-x+x^2y^2}$ (Ans)

$\left[\because \tan(x+y) = xy,\right.$

$\left.\therefore \sec^2(x+y) = 1+\tan^2(x+y) = 1+x^2y^2\right]$

$y = e^{y/x}$

Taking log, $\log y = \frac{y}{x} \log e$

or, $\log y = \frac{y}{x}$ ---- (i)

or, $x \log y = y$.

Diff, $\frac{x}{y} \frac{dy}{dx} + \log y \cdot 1 = \frac{dy}{dx}$

or, $\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -\frac{y}{x}$ [From (i)]

or, $\frac{dy}{dx}\left(\frac{x-y}{y}\right) = -\frac{y}{x}$

or, $\frac{dy}{dx} = \frac{y^2}{x(y-x)}$

$$ye^y = x$$

$$ye^y = x$$

Taking log, $\log y + y = \log x$

Diff. w.r.t. x, $\frac{d}{dx}(\log y + y) = \frac{d}{dx}(\log x)$

or, $\frac{1}{y}\frac{dy}{dx} + \frac{dy}{dx} = \frac{1}{x}$

or, $\frac{dy}{dx}\left(\frac{1}{y} + 1\right) = \frac{1}{x}$

or, $\frac{dy}{dx}\left(\frac{1+y}{y}\right) = \frac{1}{x}$

or, $\frac{dy}{dx} = \frac{y}{x(1+y)}$ [Ans].

$$x^{\sin y} + y^{\sin x} = 1$$

GOOD BOY
PAGE NO:
DATE:

$$x^{\sin y} + y^{\sin x} = 1$$

Diff. w.r.t. x, $\frac{d}{dx}\left(x^{\sin y}\right) + \frac{d}{dx}\left(y^{\sin x}\right) = \frac{d}{dx}(1)$

or, $\frac{du}{dx} + \frac{dv}{dx} = 0$ ---- (i) $\left[\text{let, } u = x^{\sin y} \text{ \& } v = y^{\sin x}\right]$

Now, $u = x^{\sin y}$

taking log, $\log u = \log x^{\sin y}$

or, $\log u = \sin y . \log x$

Diff. w.r.t. x, $\frac{d}{dx}(\log u) = \frac{d}{dx}(\sin y . \log x)$

or, $\frac{d}{du}(\log u)\frac{du}{dx} = \sin y . \frac{1}{x} + \log x . \cos y \frac{dy}{dx}$

or, $\frac{1}{u}\frac{du}{dx} = \frac{\sin y}{x} + \log x . \cos y . \frac{dy}{dx}$

or, $\frac{du}{dx} = x^{\sin y}\left(\frac{\sin y}{x} + \log x . \cos y . \frac{dy}{dx}\right)$.

$v = y^{\sin x}$

taking log, $\log v = \sin x . \log y$

diff. w.r.t x, $\frac{d}{dx}(\log v) = \frac{d}{dx}(\sin x . \log y)$

or, $\frac{d}{dv}(\log v) . \frac{dv}{dx} = \frac{\sin x}{y}\frac{dy}{dx} + \log y . \cos x$

or, $\frac{1}{v}\frac{dv}{dx} = \frac{\sin x}{y}\frac{dy}{dx} + \log y . \cos x$

or, $\frac{dv}{dx} = y^{\sin x}\left(\frac{\sin x}{y}\frac{dy}{dx} + \log y . \cos x\right)$

From (i),

$\frac{x^{\sin y} . \sin y}{x} + x^{\sin y} . \log x . \cos y \frac{dy}{dx}$

$+ \frac{y^{\sin x} . \sin x}{y}\frac{dy}{dx} + y^{\sin x} . \log y . \cos x = 0$

or, $\left(x^{\sin y} . \log x . \cos y + \frac{y^{\sin x} . \sin x}{y}\right)\frac{dy}{dx}$

$= -y^{\sin x} . \log y . \cos x - \frac{x^{\sin y} . \sin y}{x}$

or, $\frac{1}{y}\left(y \log x \cos y . x^{\sin y} + y^{\sin x} \sin x\right)\frac{dy}{dx}$

$= -\frac{1}{x}\left(x \log y \cos x . y^{\sin x} + x^{\sin y} \sin y\right)$

or, $\frac{dy}{dx} = -\frac{y}{x}\frac{\left(x^{\sin y} \sin y + x y^{\sin x} \log y \cos x\right)}{\left(x^{\sin y} y \log x \cos y + y^{\sin x} \sin x\right)}$

$y = x^{\boxed{x^{x^{\cdots\infty}}}} \approx y$

PAGE NO:

DATE:

$\therefore y = x^y$

Taking log, $\log y = y \log x$

Diff. w.r.t x, $\frac{d}{dx}(\log y) = \frac{d}{dx}(y \log x)$

or, $\frac{d}{dy}(\log y)\frac{dy}{dx} = \frac{dy}{dx}.\log x + \frac{y}{x}$

or, $\frac{1}{y}\frac{dy}{dx} = \log x.\frac{dy}{dx} + \frac{y}{x}$

or, $\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

or, $\frac{dy}{dx} = \frac{y/x}{\frac{1}{y} - \log x} = \frac{y^2}{x(1 - y\log x)}$

$$y = x + \cfrac{1}{x + \cfrac{1}{x + \cdots\infty}}$$

$$y = x + \cfrac{1}{\boxed{x + \cfrac{1}{x + \cdots\infty}}} = y$$

$$y = x + \frac{1}{y} \Rightarrow y^2 = xy + 1,$$

Diff. w.r.t. x, $\frac{d}{dx}(y^2) = \frac{d}{dx}(xy) + \frac{d}{dx}(1)$

or, $\frac{d}{dy}(y^2)\frac{dy}{dx} = x\frac{dy}{dx} + y.1 + 0$

or, $2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

or, $\frac{dy}{dx}(2y - x) = y$ or, $\frac{dy}{dx} = \frac{y}{2y - x}$

Solutions of 2:

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then show that $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

GOOD BOY

PAGE NO:

DATE:

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ $[a=\text{const}]$.

then show that, $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Let, $x=\sin\theta$, $y=\sin\phi$.

$\because \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$\Rightarrow \cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$

$\Rightarrow \cancel{2}\cos\cancel{\frac{\theta+\phi}{2}}\cos\frac{\theta-\phi}{2}=a.\cancel{2}\cos\cancel{\frac{\theta+\phi}{2}}\sin\frac{\theta-\phi}{2}$

$\Rightarrow \frac{1}{a}=\tan\frac{\theta-\phi}{2} \Rightarrow \frac{\theta-\phi}{2}=\tan^{-1}\left(\frac{1}{a}\right)$

$\Rightarrow \theta-\phi=2\tan^{-1}\left(\frac{1}{a}\right)$.

~~Diff. w.r.t.~~ $\therefore \sin^{-1}x-\sin^{-1}y=2\tan^{-1}\left(\frac{1}{a}\right)$

Diff. w.r.t. x, $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$ $[\because a=\text{const}]$

$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ (Proved).

If $y = \frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$ then show that $\frac{dy}{d\theta} + \frac{1}{1-\cos\theta} = 0$

GOOD BOY
PAGE NO:
DATE:

$$y = \frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$$

$$\Rightarrow y = \frac{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\left[\begin{aligned}\cos\theta &= 2\cos^2\frac{\theta}{2} - 1 \\ &= 1 - 2\sin^2\frac{\theta}{2}\end{aligned}\right]$$

$$\Rightarrow y = \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}$$

$$\Rightarrow y = \cot\frac{\theta}{2}$$

$$\therefore \frac{dy}{d\theta} = -\operatorname{cosec}^2\frac{\theta}{2}\cdot\frac{1}{2} = -\frac{1}{2\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{dy}{d\theta} = -\frac{1}{1-\cos\theta} \Rightarrow \frac{dy}{d\theta} + \frac{1}{1-\cos\theta} = 0$$

(proved)

If $\sin y = x\sin(a+y)$ then prove that $\frac{dy}{dx} = \frac{\sin a}{1-2x\cos a + x^2}$

$\sin y = x\sin(a+y)$

$\Rightarrow \sin y = x(\sin a \cos y + \cos a \sin y)$

$\Rightarrow \sin y = x\sin a \cos y + x\cos a \sin y$

$\Rightarrow \sin y - x\cos a \sin y = x\sin a \cos y$

$\Rightarrow (1 - x\cos a)\sin y = x\sin a \cos y$

$$\Rightarrow \quad \tan y = \frac{x\sin a}{1 - x\cos a} \quad \text{---- (i)}$$

Diff. w.r.t. x,

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}\left\{\frac{x\sin a}{(1 - x\cos a)}\right\}$$

PAGE NO:

DATE:

$$\Rightarrow \sec^2 y \frac{dy}{dx} = \frac{(1-x\cos a)\frac{d}{dx}(x\sin a) - x\sin a\frac{d}{dx}(1-x\cos a)}{(1-x\cos a)^2}$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} = \frac{(1-x\cos a)\sin a - x\sin a(-\cos a)}{(1-x\cos a)^2}$$

$$= \frac{\sin a - x\sin a\cancel{\cos a} + x\sin a\cancel{\cos a}}{(1-x\cos a)^2} \quad \text{---(ii)}$$

Now,

$$\sec^2 y = 1 + \tan^2 y = 1 + \left(\frac{x\sin a}{1-x\cos a}\right)^2 \quad [\text{From (i)}]$$

$$\text{or, } \sec^2 y = \frac{(1-x\cos a)^2 + x^2\sin^2 a}{(1-x\cos a)^2}$$

$$\text{or, } \sec^2 y = \frac{1-2x\cos a + x^2\cos^2 a + x^2\sin^2 a}{(1-x\cos a)^2}$$

$$\text{or, } \sec^2 y = \frac{1-2x\cos a + x^2}{(1-x\cos a)^2}$$

From (ii), putting value of $\sec^2 y$,

$$\frac{(1-2x\cos a + x^2)}{(1-x\cos a)^2}\cdot\frac{dy}{dx} = \frac{\sin a}{(1-x\cos a)^2}$$

or,

$$\frac{dy}{dx} = \frac{\sin a}{1-2x\cos a + x^2} \quad \text{(proved)}$$

If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ then show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

GOOD BOY
PAGE NO:
DATE:

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$$

Diff. w.r.t. x, we get,

$$\sqrt{x^2+1}\cdot\frac{dy}{dx} + y\cdot\frac{d}{dx}(x^2+1)^{1/2} = \frac{d}{dx}\log\left(\sqrt{x^2+1} - x\right)$$

$$\text{or, } \sqrt{x^2+1}\,\frac{dy}{dx} + y\cdot\frac{2x}{2\sqrt{x^2+1}} = \frac{\frac{2x}{2\sqrt{x^2+1}} - 1}{\left(\sqrt{x^2+1} - x\right)}$$

$$\text{or, } \sqrt{x^2+1}\,\frac{dy}{dx} + \frac{xy}{\sqrt{x^2+1}} = \frac{\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}}{\left(\sqrt{x^2+1} - x\right)}$$

$$\text{or, } \sqrt{x^2+1}\,\frac{dy}{dx} + \frac{xy}{\sqrt{x^2+1}} = -\frac{1}{\sqrt{x^2+1}}$$

$$\text{or, } (x^2+1)\frac{dy}{dx} + xy + 1 = 0 \quad \text{(proved)}$$

If $y = 1 + \frac{a}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$ then show that $\frac{dy}{dx} = \frac{y}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$

$$y = \left(1 + \frac{a}{x-a}\right) + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$

$$\text{or, } y = \frac{x-a+a}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$

$$\text{or, } y = \left(\frac{x}{x-a} + \frac{bx}{(x-a)(x-b)}\right) + \frac{cx^2}{(x-a)(x-b)(x-c)}$$

$$\text{or, } y = \frac{x(x-b)+bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$

$$\text{or, } y = \frac{x^2}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$

GOOD BOY
PAGE NO:
DATE:

or, $y = \dfrac{x^2(x-c)+cx^2}{(x-a)(x-b)(x-c)} = \dfrac{x^3}{(x-a)(x-b)(x-c)}$

Taking log,

$\log y = 3\log x - \log(x-a) - \log(x-b) - \log(x-c)$.

Diff. w.r.t. x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} - \frac{1}{x-a} - \frac{1}{x-b} - \frac{1}{x-c}$$

$$\text{or, } \frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{x} - \frac{1}{x-a}\right) + \left(\frac{1}{x} - \frac{1}{x-b}\right) + \left(\frac{1}{x} - \frac{1}{x-c}\right)$$

$$\text{or, } \frac{1}{y}\frac{dy}{dx} = \frac{\cancel{x}-a-\cancel{x}}{x(x-a)} + \frac{\cancel{x}-b-\cancel{x}}{x(x-b)} + \frac{\cancel{x}-c-\cancel{x}}{x(x-c)}$$

$$\text{or, } \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$$

$$\text{or, } \frac{dy}{dx} = \frac{y}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$$

$\boxed{\text{Proved}}$

If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

$x = \sec\theta - \cos\theta$.
$y = \sec^n\theta - \cos^n\theta$.

Let, $\sec\theta = t$, $\therefore \cos\theta = \frac{1}{t}$

$\therefore x = t - \frac{1}{t}$, $y = t^n - \frac{1}{t^n}$.

$\frac{dx}{dt} = 1 + \frac{1}{t^2} = \frac{1}{t}\left(t + \frac{1}{t}\right)$

$\frac{dy}{dt} = nt^{n-1} - (-n)t^{-n-1} = nt^{n-1} + \frac{n}{t^{n+1}}$

or, $\frac{dy}{dt} = n\left(\frac{t^n}{t} + \frac{1}{t.t^n}\right) = \frac{n}{t}\left(t^n + \frac{1}{t^n}\right)$

$\therefore \frac{dy}{dx} = \frac{\frac{n}{t}\left(t^n + \frac{1}{t^n}\right)}{\frac{1}{t}\left(t + \frac{1}{t}\right)}$

or, $\left(\frac{dy}{dx}\right)^2 = \frac{n^2\left(t^n + \frac{1}{t^n}\right)^2}{\left(t + \frac{1}{t}\right)^2} = \frac{n^2\left\{\left(t^n - \frac{1}{t^n}\right)^2 + 4\right\}}{\left\{\left(t - \frac{1}{t}\right)^2 + 4\right\}}$

or, $\left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{(x^2+4)}$ or, $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

Proved

If $f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2x}$ then show that $f'(0)=\left(2\log\frac{a}{b}+\frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$

$$f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2x}$$

Taking log, $\log f(x)=(a+b+2x)\{\log(a+x)-\log(b+x)\}$

Diff. w.r.t. x,

$$\frac{1}{f(x)}\cdot f'(x)=\frac{(a+b+2x)}{a+x}+\log(a+x).2-\frac{(a+b+2x)}{b+x}-\log(b+x)\cdot 2$$

Now, putting $x=0$, $\left[f(0)=\left(\frac{a}{b}\right)^{a+b}\right]$

we get,

$$f'(0)=\left(\frac{a}{b}\right)^{a+b}\left[\frac{a+b}{a}+2\log a-\frac{a+b}{b}-2\log b\right]$$

$$=\left(\frac{a}{b}\right)^{a+b}\left[\frac{ab+b^2-a^2-ab}{ab}+2\log\left(\frac{a}{b}\right)\right]$$

or, $f'(0)=\left(\frac{a}{b}\right)^{a+b}\left[\frac{b^2-a^2}{ab}+2\log\left(\frac{a}{b}\right)\right]$

Proved