Determinant

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1) If $A = \begin{bmatrix} 11 & 20 \\ 31 & 41 \end{bmatrix}$ and $|3A| = k|A|$, then find the value of k, where $|A|$ is determinant of matrix A.

2) Let A be a square matrix of order $n \times n$, and k is a scalar, then $|kA|$ is equal to -- (a) $k|A|$ (b) $nk|A|$ (c) $n^k|A|$ (d) $k^n|A|$

3) Find co-factor of -4 and 9 in determinant $\begin{vmatrix} -1 & -2 & 3 \\ -4 & -5 & -6 \\ -7 & 8 & 9 \end{vmatrix}$

4) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -i \\ 20 & 3 & i \end{vmatrix} = x + iy$, then find the values of x & y where $i = \sqrt{-1}$

5) Value of the determinant $\begin{vmatrix} 2^{10} & 2^{11} & 2^{12} \\ 2^{11} & 2^{12} & 2^{13} \\ 2^{12} & 2^{13} & 2^{14} \end{vmatrix}$ is -- (a) 2^{10} (b) 0 (c) 1 (d) None of these.

6) Find the value of $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

7) If A is a skew-symmetric matrix of odd order, then value of $|A|$ is -- (a) 1 (b) -1 (c) 0 (d) any real number.

8) If A is an invertible matrix of order 2, then the value of $\det\left(A^{-1}\right)$ is -- (a) 0 (b) 1 (c) $\det A$ (d) $\dfrac{1}{\det A}$

9) If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$ then the value of $|A| \cdot |adj(A)|$ is -- (a) x^9 (b) x^6 (c) x^3 (d) x

10) A be a 3×3 matrix. If A^{-1} exists and $|A| = 6$, then the value of $|Adj(A)|$ is -- (a) 1 (b) 6 (c) 12 (d) 36

11) If $A = \left[a_{ij}\right]_{n \times n}$ and $|A| = d \neq 0$, then $|adj\ A|$ is -- (a) d (b) d^n (c) d^{n-1} (d) d^{n+1}

12) If A is an orthogonal matrix of order 3, then value of $|A|$ is -- (a) 0 (b) 1 (c) -1 (d) ± 1

13) The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if a, b, c are in -- (a) A.P. (b) G.P. (c) H.P. (d) None of these.

14) Prove that $\begin{vmatrix} a & b & a\alpha+b\beta \\ b & c & b\alpha+c\beta \\ a\alpha+b\beta & b\alpha+c\beta & 0 \end{vmatrix} = (b^2-ac)(a\alpha^2+2b\alpha\beta+c\beta^2)$

15) Without expanding, prove that: $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

16) If $\begin{vmatrix} -5 & 5 & 10 \\ 5 & -5 & x \\ 0 & 10 & 5 \end{vmatrix} = 0$ then find x

17) Solve: $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

18) Solve : $\begin{vmatrix} x+4 & 3 & 5 \\ 3 & x+4 & 5 \\ 3 & 5 & x+4 \end{vmatrix} = 0$

19) Solve for x. $\begin{vmatrix} x+2 & 3 & 3 \\ 3 & x+4 & 5 \\ 3 & 5 & x+4 \end{vmatrix} = 0$

20) Solve: $\begin{vmatrix} 2-x & 2 & 3 \\ 2 & 5-x & 6 \\ 3 & 4 & 10-x \end{vmatrix} = 10$

21) Solve: $\begin{vmatrix} 11-x & -6 & 2 \\ -6 & 10-x & -4 \\ 2 & -4 & 6-x \end{vmatrix} = 0$

22) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$ then find the roots of $\det\,(A - xI_3) = 0$ where I is the unit matrix of order 3.

23) If $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x+1 & 1 \\ -1 & 1 & x+1 \end{vmatrix} = 0$, then show that values of x are $0, 1, -2$.

24) Solve: $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = 0$

25) Solve : $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = 0$

26) Solve for x: $\begin{vmatrix} x & c+x & b+x \\ c+x & x & a+x \\ b+x & a+x & x \end{vmatrix} = 0$

27) Solve: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

28) Solve : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

29) Solve: $\begin{vmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 16 & 64 \end{vmatrix} = 0$

30) Solve. $\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$

31) Without expanding prove that: $\begin{vmatrix} bc & a^2 & a^2 \\ b^2 & ca & b^2 \\ c^2 & c^2 & ab \end{vmatrix} = \begin{vmatrix} bc & ab & ca \\ ab & ca & bc \\ ca & bc & ab \end{vmatrix}$

32) Without expanding, prove that: $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

33) Without expanding, show that: $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

34) Without expanding, prove that: $\begin{vmatrix} \frac{1}{a} & 1 & bc \\ \frac{1}{b} & 1 & ca \\ \frac{1}{c} & 1 & ab \end{vmatrix} = 0$

35) Without expanding prove that $\begin{vmatrix} 5 & 2 & 3 \\ 7 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

36) Without expanding prove that: $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 0$

37) Without expanding prove that: $\begin{vmatrix} 27 & 40 & 58 \\ 24 & 36 & 52 \\ 18 & 28 & 40 \end{vmatrix} = 0$

38) Without expanding, prove that: $\begin{vmatrix} 101 & 103 & 105 \\ 104 & 105 & 106 \\ 107 & 108 & 109 \end{vmatrix} = 0$

39) Without expanding, prove that: $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 448 & 449 & 450 \end{vmatrix} = 0$

40) Without expanding prove that $\begin{vmatrix} 9 & 9 & 12 \\ 1 & -3 & -4 \\ 1 & 9 & 12 \end{vmatrix} = 0$

41) Without expanding, prove that: $\begin{vmatrix} 0 & 2016 & -2017 \\ -2016 & 0 & 2018 \\ 2017 & -2018 & 0 \end{vmatrix} = 0$

42) Without expanding prove that: $\begin{vmatrix} a+1 & a+4 & a+2 \\ a+2 & a+5 & a+4 \\ a+3 & a+6 & a+6 \end{vmatrix} = 0$

43) Without expanding prove that: $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix} = 0$

44) Show that the value of $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+5 & x+8 & x+12 \end{vmatrix}$, is not depend on x.

45) If a, b, c are in Arithmetic Progression (A.P.) , then show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

46) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ C_1^m & C_1^{m+1} & C_1^{m+2} \\ C_2^m & C_2^{m+1} & C_2^{m+2} \end{vmatrix} = 1$

47) Prove that $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = 0$

48) Without expanding prove that: $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix} = 0$

49) If p^{th}, q^{th} and r^{th} terms of a geometric series are a, b, c respectively, then show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$ where a, b, c are positive.

50) Find the value of $\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}$

51) Find the value of: $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

52) Without expanding, prove that $\begin{vmatrix} 3 & 4 & 5 \\ 4 & 3 & 7 \\ 5 & 7 & 5 \end{vmatrix}$ is divisible by 23.

53) Find the value (without expanding) $\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$

54) Find the value (without expanding): $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

55) If $x = -9$ is one root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then find other two roots.

56) If $\begin{vmatrix} a & b & c \\ x & y & z \\ m & n & p \end{vmatrix} = \lambda$, then find the value of $\begin{vmatrix} 6a & 2b & 2c \\ 3x & y & z \\ 3m & n & p \end{vmatrix}$ in terms of λ

57) If $\begin{vmatrix} x & z & y \\ a & c & b \\ 1 & 1 & 1 \end{vmatrix} = 7$, then show that $\begin{vmatrix} c-b & y-z & zb-yc \\ b-a & x-y & ay-xb \\ a-c & z-x & cx-az \end{vmatrix} = 49$

58) Find the value of $\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$ where ω is the imaginary cube root of unity.

59) Find the value of $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$

60) Find the value of $\begin{vmatrix} 1+2\omega^{100}+\omega^{200} & \omega^2 & 1 \\ 1 & 1+\omega^{100}+2\omega^{200} & \omega \\ \omega & \omega^2 & 2+\omega^{100}+\omega^{200} \end{vmatrix}$ where ω is the imaginary cube root of unity.

61) If α, β are imaginary roots of $x^3 - 1 = 0$, then show that $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix} = \lambda^3$

62) If a, b, c are non-zero unequal real numbers and $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, then prove that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

63) Without expanding prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

64) Prove that $\begin{vmatrix} 1 & x & x^2-yz \\ 1 & y & y^2-zx \\ 1 & z & z^2-xy \end{vmatrix} = 0$

65) Without expanding prove that: $\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

66) Without expanding prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

67) without expanding, prove that: $\begin{vmatrix} a^3 & a^2 & 1 \\ b^3 & b^2 & 1 \\ c^3 & c^2 & 1 \end{vmatrix} = (ab+bc+ca)\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$

68) Without expanding, prove that: $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

69) Prove without expanding. $\begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$

70) Without expanding prove that: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

71) without expanding, prove that: $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix} = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

72) Prove that $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} = 2 \times 10! \times 11! \times 12!$

73) Show that $\begin{vmatrix} a^2 & 2a & 1 \\ 1 & a^2 & 2a \\ 2a & 1 & a^2 \end{vmatrix}$ is a perfect square quantity.

74) Prove that $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3-1)^2$

75) Prove that $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix} = -(a^3+b^3)^2$

76) Prove $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = a^3+b^3$. Hence find the value of $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix}$

77) Find the value (without expanding). $\begin{vmatrix} a^{-1} & a^2 & bc \\ b^{-1} & b^2 & ca \\ c^{-1} & c^2 & ab \end{vmatrix}$

78) If $\Delta = \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ and $a+b+c=0$, then show that either $x=0$ or $x = \pm\sqrt{\frac{3}{2}\left(a^2+b^2+c^2\right)}$

79) Prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$ where ω is the imaginary cube root of unity.

80) Prove that $f(200)=0$ where $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

81) Prove that: $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

82) Prove that: $\begin{vmatrix} x^2+y^2+1 & x^2+2y^2+3 & x^2+3y^2+4 \\ y^2+2 & 2y^2+6 & 3y^2+8 \\ y^2+1 & 2y^2+3 & 3y^2+4 \end{vmatrix} = x^2y^2$

83) Prove without expanding. $\begin{vmatrix} \cos(x-\alpha) & \cos(x+\alpha) & cosx \\ \sin(x+\alpha) & \sin(x-\alpha) & sinx \\ \cos\alpha\, tanx & \cos\alpha\, cotx & \csc\ 2x \end{vmatrix} = 0$

84) Prove that: $\begin{vmatrix} \cos(x+y) & \sin(x+y) & -\cos(x+y) \\ \sin(x-y) & \cos(x-y) & \sin(x-y) \\ \sin 2x & 0 & \sin 2y \end{vmatrix} = \sin 2(x+y)$

85) Prove that $\begin{vmatrix} 2\cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 0 & 1 & 2\cos\theta \end{vmatrix} = \dfrac{\sin 4\theta}{\sin\theta}$ $[\theta \neq n\pi]$

86) If $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ 1 & -\sin\theta & 1 \end{vmatrix}$ then show that $2 \leq \Delta \leq 4$

87) Show that $\begin{vmatrix} 1 & 1 & 1 \\ \sin\alpha & \sin\beta & \sin\gamma \\ \cos\alpha & \cos\beta & \cos\gamma \end{vmatrix} = -4\sin\dfrac{\alpha-\beta}{2}\sin\dfrac{\beta-\gamma}{2}\sin\dfrac{\gamma-\alpha}{2}$

88) Show that $\begin{vmatrix} \sin^2 A & sinA & \cos^2 A \\ \sin^2 B & sinB & \cos^2 B \\ \sin^2 C & sinC & \cos^2 C \end{vmatrix} = (\sin A - \sin B)(\sin B - \sin C)(\sin A - \sin C)$

89) If A, B, C are three angles of a triangle, then prove that $\begin{vmatrix} -1 & \cos\ C & \cos\ B \\ \cos\ C & -1 & \cos\ A \\ \cos\ B & \cos\ A & -1 \end{vmatrix} = 0$

90) If A, B, C are three angles of a triangle, then prove that
$\begin{vmatrix} \sin\ (A+B+C) & \sin\ (A+C) & \cos\ C \\ -\sin\ B & 0 & \tan\ A \\ \cos\ (A+B) & \tan\ (B+C) & 0 \end{vmatrix} = 0$

91) Prove that $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & y+x \end{vmatrix} = 4xyz$

92) Show that (without expanding) $\begin{vmatrix} 1 & x^2+yz & x^3 \\ 1 & y^2+zx & y^3 \\ 1 & z^2+xy & z^3 \end{vmatrix} = -(x-y)(y-z)(z-x)(x^2+y^2+z^2)$

93) If c is any constant, then prove that $\begin{vmatrix} x & x^2 & 1+cx^3 \\ y & y^2 & 1+cy^3 \\ z & z^2 & 1+cz^3 \end{vmatrix} = (1+cxyz)(x-y)(y-z)(z-x)$

94) If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1+xyz = 0$

95) Prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

96) Prove that $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix} = \lambda^2(a^2+b^2+c^2+\lambda)$

97) Show that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

98) Show that $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ca & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

99) Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

100) Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

101) Show that

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

102) Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\ (a+b+c)^3$

103) Prove that $\begin{vmatrix} 2a & a-b-c & 2a \\ 2b & 2b & b-c-a \\ c-a-b & 2c & 2c \end{vmatrix} = (a+b+c)^3$

104) Prove that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

105) Prove that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2\ (a+b)\ (b+c)\ (c+a)$

106) Prove (without expanding). $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\ (a+b+c)^3$

107) Without expanding, prove that: $\begin{vmatrix} (a+b)^2 & ac & bc \\ ac & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3$

108) If $a^2 + b^2 + c^2 = 0$ and $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$, then find the value of k.

109) If $2s = a + b + c$ then prove that
$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$

110) Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

111) Prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

112) Prove that: $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

113) Prove that $\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

114) Prove that: $\begin{vmatrix} 1 & b+c & c^2 \\ 1 & c+a & a^2 \\ 1 & a+b & b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

115) Prove that $\begin{vmatrix} -1 & b & c \\ a & -1 & c \\ a & b & -1 \end{vmatrix} = (a+1)(b+1)(c+1)(\frac{a}{a+1} + \frac{b}{b+1} + \frac{c}{c+1} - 1)$

116) Prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xy + yz + zx + xyz = xyz\,(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z})$

117) Without expanding prove that: $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix}$

$$= (x-a)(y-b)(z-c)\left(\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c} - 2\right)$$

118) If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then show that $\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c} = 2.$

119) If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$, then show that $\dfrac{p}{x} + \dfrac{q}{y} + \dfrac{r}{z} = 2$

120) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

121) If $x^3 = 1$, then prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a + bx + cx^2)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$

122) Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - (a^3 + b^3 + c^3)$

123) Prove that: $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

124) If a, b, c are real numbers, then prove that
$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -2(a^3 + b^3 + c^3 - 3abc)$ and hence show that if the

value of this determinant is zero, then either $a+b+c=0$ or $a=b=c$.

125) Prove that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$

126) Using properties of determinant, prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

127) Justify the truth. $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

128) Prove that $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$

129) If $S_n = \alpha^n + \beta^n + \gamma^n$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$

130) Show that the condition for which equations $x+ay+az=0$, $bx+y+bz=0$, $cx+cy+z=0$ have non-zero solution is $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c} = -1$

131) Show that the equations $(\lambda+a)x+\lambda y+\lambda z=0$, $\lambda x+(\lambda+b)y+\lambda z=0$ and $\lambda x+\lambda y+(\lambda+c)z=0$ have non-zero solutions if $\frac{1}{\lambda} = -\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

132) Solve by Cramer's rule: $3x+y+z=2$, $2x-4y+3z=-1$, $4x+y-3z=-11$

133) Solve by Cramer's rule: $x-2y+z=-1$, $3x+y-2z=4$, $y-z=1$

134) Solve by Cramer's rule: $2x-y+z=6$, $x+2y+3z=3$, $3x+y-z=4$

135) Solve by Cramer's rule: $x+y+z=1$, $ax+by+cz=k$, $a^2x+b^2y+c^2z=k^2$ where [$a \neq b \neq c$]

136) Solve by Cramer's rule: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, $\frac{2}{x}+\frac{5}{y}+\frac{3}{z}=0$, $\frac{1}{x}+\frac{2}{y}+\frac{4}{z}=0$

137) Solve by Cramer's rule: $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}=\frac{1}{2}$, $\frac{4}{x}+\frac{2}{y}-\frac{3}{z}=\frac{2}{3}$, $\frac{3}{x}-\frac{4}{y}+\frac{4}{z}=\frac{1}{3}$

138) If the points (a_1, b_1), (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear, then show that $a_1b_2 = a_2b_1$

139) If the coordinates of vertices of a triangle are $[m(m+1), m+1]$, $[(m+1)(m+2), m+2]$ and $[(m+2)(m+3), m+3]$, then prove that the area of said triangle is not depend on m.