# Derivative (2nd order)

For answers & solutions, visit: this link (https://jsarkar.com/chapter/2nd-order-derivative)

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1) If $x^2 + y^2 = 25$ then find the value of $\frac{d^2y}{dx^2}$ at $x = 0$

2) If $x = a\cos nt - b\sin nt$, then find the value of $\frac{d^2x}{dt^2}$

3) If $y = \frac{1}{2}x\frac{dy}{dx}$ then show that $\frac{d^2y}{dx^2}$ is constant

4) If $2y = x\left(1 + \frac{dy}{dx}\right)$, then show that $\frac{d^2y}{dx^2}$ is constant.

5) If $y = \sin 3x \cos 4x$ then find $\frac{d^2y}{dx^2}$

6) If $y = \tan^{-1}(\sec x + \tan x)$ then find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$

7) If $\frac{dx}{dy} = u$ and $\frac{d^2x}{dy^2} = v$, then show that $\frac{d^2y}{dx^2} = -\frac{v}{u^3}$

8) If $pv^a = c$ where $a$, $c$ are constants, then show that $v^2\frac{d^2p}{dv^2} = a(a+1)p$

9) If $V = \frac{A}{r} + B$ (where $A$, $B$ are constants) then show that $\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0$

10) If $y = \frac{\log x}{x}$ then find the value of $\left(\frac{d^2y}{dx^2}\right)_{x=1}$

11) If $y = e^{\frac{1}{x}}$, then find $\frac{d^2y}{dx^2}$

12) If $F(x) = f(x)\cdot\phi(x)$ and $f'(x)\cdot\phi'(x) = a$ where $a$ constant, then prove that $\frac{F''}{F} = \frac{f''}{f} + \frac{\phi''}{\phi} + \frac{2a}{f\phi}$

13) If $x = f(t)$ and $y = g(t)$ then show that $\frac{d^2y}{dx^2} = \frac{x_1y_2 - y_1x_2}{x_1^3}$

14) If $x \sin\theta + y\cos\theta = a$ and $x\cos\theta - y\sin\theta = b$, then show that value of $\frac{dx}{d\theta} \cdot \frac{d^2y}{d\theta^2} - \frac{d^2x}{d\theta^2} \cdot \frac{dy}{d\theta}$ is constant.

15) If $x = t^2 + 2t$, $y = t^3 - 3t$ then show that $\frac{d^2y}{dx^2} = \frac{3}{4(t+1)}$

16) If $x = \cos t$, $y = \log t$ then show that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ at $t = \frac{\pi}{2}$

17) If $x = a\cos 2t$, $y = a\sin 2t$ then find the value of $\frac{d^2y}{dx^2}$ in form of $t$

18) If $x = \sin t$, $y = \sin kt$ then show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$ where $k$ is a constant

19) If $x = \tan t$ and $y = \tan pt$, then show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + 2(x - py)\frac{dy}{dx} = 0$

20) If $x = e^t \sin t$ and $y = e^t \cos t$ then show that $(x+y)^2\frac{d^2y}{dx^2} = 2\left(x\frac{dy}{dx} - y\right)$

21) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ then the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

22) If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

23) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ where $a$ is constant. then prove that $2y\frac{d^2y}{dx^2} + \csc^2\frac{\theta}{2} = 0$ [csc is abbreviation of cosec]

24) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

25) If $x = a\sin 2\theta(1 + \cos 2\theta)$ and $y = a\cos 2\theta(1 - \cos 2\theta)$, then prove that $1 + \left(\frac{dy}{dx}\right)^2 = \sec^2\theta$. Then show that $4a\cos 3\theta\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$

26) If $x = a\cot\theta$, $y = \frac{1}{x^2 + a^2}$, then show that $\frac{d^2y}{dx^2} = \frac{2}{a^4}\sin^3\theta\sin 3\theta$

27) If $y = e^x(\sin x + \cos x)$ then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

28) If $y = x^3 \log \frac{1}{x}$ then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

29) If $y = x\log\left(\frac{x}{a+bx}\right)$ then show that $x^3\frac{d^2y}{dx^2} = \left(y - x\frac{dy}{dx}\right)^2$

30) If $x + y = e^{x-y}$ then show that $\frac{d^2y}{dx^2} = \frac{4(x+y)}{(x+y+1)^3}$

31) If $x^2 + y^2 = a^2$ then show that $\frac{\left(1+y_1^2\right)^{\frac{3}{2}}}{y_2} = -a$ where $a$ is constant and $y_1$, $y_2$ are first & second order derivative

32) If $y^{\frac{1}{3}} + y^{-\frac{1}{3}} = 2x$, then prove that $\left(x^2-1\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 9y$

33) If $2x = y^{\frac{1}{m}} + y^{-\frac{1}{m}}$, then show that $\left(x^2-1\right)y_2 + xy_1 = m^2y$

34) If $(a+bx)e^{\frac{x}{y}} = x$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

35) If $y = \frac{1}{1+x+x^2+x^3}$ then find the value of $\left[\frac{dy}{dx}\right]_{x=0}$ and $\left[\frac{d^2y}{dx^2}\right]_{x=0}$

36) If $ax^2 + 2hxy + by^2 = 1$, then show that $\frac{d^2y}{dx^2} = \frac{h^2-ab}{(hx+by)^3}$

37) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then show that $(1+x)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = \frac{2}{(1+x)^3}$

38) If $\sqrt{y+x} + \sqrt{y-x} = c$ ($c$ is a non-zero constant), then show that $\frac{d^2y}{dx^2} = \frac{2}{c^2}$

39) If $y = \sqrt{x+1} - \sqrt{x-1}$, then show that $\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{1}{4}y$

40) If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ then show that $\left(1 - x^2\right)y_2 - xy_1 + m^2 y = 0$

41) If $y = \left(x + \sqrt{1 + x^2}\right)^m$ then show that $\left(1 + x^2\right)y_2 + xy_1 - m^2 y = 0$. Then find the value of $(y_2)_0$

42) If $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$, then show that $\left(1 + x^2\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 2$

43) If $y = A\left(x + \sqrt{x^2 - 1}\right)^n + B\left(x - \sqrt{x^2 - 1}\right)^n$ then prove that $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + x^2 y = 0$

44) If $y = \cos\left[m \log_e\left(x + \sqrt{x^2 + 1}\right)\right]$, then show that $\left(x^2 + 1\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + m^2 y = 0$

45) If $\sin x + \cos y = 1$ then show that $\dfrac{d^2y}{dx^2} = -\dfrac{\sin^2 x + \cos y}{\sin^3 y}$

46) If $p^2 = a^2 \cos^2\theta + b^2 \sin^2\theta$ then show that $p + \dfrac{d^2p}{d\theta^2} = \dfrac{a^2 b^2}{p^3}$

47) If $y = \sin(\sin x)$, then show that $\dfrac{d^2y}{dx^2} + \tan x \dfrac{dy}{dx} + y\cos^2 x = 0$

48) If $y = x \sin x$ then show that $x^2 y_2 - 2xy_1 + \left(2 + x^2\right)y = 0$

49) If $\log y = \sin^{-1} x$ then show that $\left(1 - x^2\right)y_2 - xy_1 - y = 0$

50) If $y = e^{m \sin^{-1} x}$ where $(-1 \le x \le 1)$, then show that $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = m^2 y$

51) If $y = e^{m \cos^{-1} x}$, then show that $\left(1 - x^2\right)y_2 - xy_1 - m^2 y = 0$

52) If $y = \sin^{-1} x$ then find the value of $\left(1 - x^2\right)y_2 - xy_1$

53) If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$, then show that $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} - y = 0$

54) If $y = \sin\left(2 \sin^{-1} x\right)$ then show that $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} = x\dfrac{dy}{dx} - 4y$

55) If $y = \sin\left(m \sin^{-1} x\right)$, then show that $\left(1 - x^2\right) y_2 - x y_1 + m^2 y = 0$

56) If $y = \cos\left(2 \sin^{-1} x\right)$ then prove that $\left(1 - x^2\right) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} + 4y = 0$

57) If $y = \cos\left(m \sin^{-1} x\right)$ then show that $\left(1 - x^2\right) y_2 - x y_1 + m^2 y = 0$

58) If $y = \left(\sin^{-1} x\right)^2$, then find the value of $\left(1 - x^2\right) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} + 4$

59) If $y = \left(\cos^{-1} x\right)^2$ then prove that $\left(1 - x^2\right) y_2 - x y_1 - 2 = 0$

60) If $y = \left(\tan^{-1} x\right)^2$ then show that $\left(1 + x^2\right)^2 y_2 + 2x\left(1 + x^2\right) y_1 - 2 = 0$

61) If $y = \left(\cot^{-1} x\right)^2$, then prove that $\left(x^2 + 1\right)^2 \dfrac{d^2y}{dx^2} + 2x\left(x^2 + 1\right) \dfrac{dy}{dx} - 2 = 0$

62) If $y = \left(\sin^{-1} x\right)^2 + \left(\cos^{-1} x\right)^2$, then show that $\left(1 - x^2\right) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} = 4$

63) If $y = a \sin^{-1} x + b \cos^{-1} x$, then prove that $\left(1 - x^2\right) y_2 - x y_1 = 0$

64) If $y = \tan^{-1}\left(\dfrac{1 - 2\log x}{1 + 2\log x}\right) + \tan^{-1}\left(\dfrac{3 + 2\log x}{1 - 6\log x}\right)$, then show that $\dfrac{d^2y}{dx^2} = 0$

65) If $y = a\cos(\log x) + b\sin(\log x)$, where $a$ & $b$ are constants, then prove that $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = 0$

66) If $y = x^{n-1} \log x$ then show that $x^2 \dfrac{d^2y}{dx^2} + (3 - 2n) x \dfrac{dy}{dx} + (n - 1)^2 y = 0$

67) If $y = \tan(x + y)$ then show that $\dfrac{d^2y}{dx^2} = \dfrac{2\left(1 + y^2\right)}{y^5}$

68) If $\sin(x + y) = kx$ where $k$ is a non-zero constant, then show that $\dfrac{d^2x}{dy^2} + x\left(\dfrac{dx}{dy} + 1\right)^3 = 0$

69) If $x = \dfrac{1}{z}$ and $y = f(x)$ then show that $\dfrac{d^2f}{dx^2} = 2z^3 \dfrac{dy}{dz} + z^4 \dfrac{d^2y}{dz^2}$

70) If $y = f(x)$ and $\log x = z$, then show that $x^2 \dfrac{d^2y}{dx^2} = \dfrac{d^2y}{dz^2} - \dfrac{dy}{dz}$

71) If $y = e^u$ and $u = f(x)$, then show that $\dfrac{d^2y}{dx^2} = e^u \left[ \dfrac{d^2u}{dx^2} + \left( \dfrac{du}{dx} \right)^2 \right]$

72) If $x = e^t$ and $\dfrac{d^2y}{dt^2} + p^2y = 0$, then show that $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + p^2y = 0$

73) If $s = \dfrac{1}{2} \log \left( \dfrac{1+t}{1-t} \right)$, then prove that $\dfrac{d^2s}{dt^2} = - \left( \dfrac{ds}{dt} \right)^3 \dfrac{d^2t}{ds^2}$